PART THREE

ESTIMATES OF MINER LOCATION ACCURACY: ERROR ANALYSIS IN SEISMIC LOCATION PROCEDURES FOR TRAPPED MINERS

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PART THREE

ESTIMATES OF MINER LOCATION ACCURACY:

ERROR ANALYSIS IN SEISMIC LOCATION PROCEDURES FOR TRAPPED MINERS

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I. SUMMARY

A method of error analysis has been applied to the location technique of non-linear least squares iterative inversion in order to evaluate the resolving power of several seismic array configurations with various assumed earth models and errors and inaccuracies in arrival times.

The results obtained demonstrate that lateral accuracies of location are improved significantly when the depth of the miner is known. Lateral location to within 100 feet appears achievable in many instances. If it is possible to refine earth models significantly beyond what has normally been assumed in this work by the use of on-site data, or if the mine is shallow (300 feet or less), accuracies of about 30 feet or so may be attainable.

Inaccuracies in earth models of about 5% are found to contribute much more heavily to these location inaccuracies than errors of a few milliseconds in picking arrival times; however, arrival time errors of 15-20 ms or above will dominate these model inaccuracies.

The expected accuracy of location is found to fall off very rapidly as the miner moves outside the array. The strength of this effect depends markedly upon the geometry of the array configuration and can be reduced by careful design. Also, as the size of the array is increased, expected location errors within the array are not altered much, but continue to match the error associated with the smaller array over a larger area (assuming that all stations can still pick up the miner's signals).

Better location accuracy, especially with respect to depth control is achievable in an earth where the velocity is depth-dependent (increasing with depth) than in one which is homogeneous. This is an advantage since the

actual earth is clearly closer to the former situation. Models where the seismic velocity increases linearly with depth can be found which are excellent approximations to a layered earth for the purposes of location.

It has to be emphasized that the location accuracies predicted in this work are subject to the major assumption that the actual behavior of the earth can be represented reasonably accurately by the models selected. If this is not the case, then location inaccuracies resulting from the use of these models may be much larger than those predicted here; new classes of models may have to be developed.

It can be concluded, however, that the location accuracies predicted for the non-linear least squares iterative inversion technique make it appear promising for use as a miner location algorithm. Alternative and potentially better location techniques remain subjects for future investigation. These may, for example, include different weighting schemes for the seismometers in an array, or allow the possibility of iterating and improving the earth model used, as well as the predicted location.

II. INTRODUCTION

Procedures for determining the location of impulsive seismic sources in the earth have been the object of studies by seismologists for many years. Recent expansion of the use of dense networks of detection stations and high frequency sensors and recording apparatus, particularly for the study of very small earthquakes, has stimulated the development of high-precision location techniques. The accuracy and resolving ability with which a given array of sensors can locate a seismic source depend on the closeness with which the model represents the real earth as well as on the array configuration; errors in the input parameters such as arrival times; and the particular algorithm used in the calculations. For the standard technique of non-linear least square iterative inversion, a method of error analysis has been developed which is very useful in evaluating the resolving power of a given station configuration with known model and arrival time errors (Peters and Crosson, 1972). The method is based on a procedure known as prediction analysis

(Wolberg, 1966), and it allows error predictions to be made without actually carrying out the inversion calculations. For the seismic location problem it is convenient to diagrammatically represent the error structure by mapping the errors onto the array geometry by means of contour maps.

The work reported in this Part is a direct application of the error analysis procedure to the problem of locating a trapped miner who is able to communicate seismically with the surface by means of producing small impulsive seismic disturbances. Given basic input data such as the arrival times of a discrete event at a series of detectors located at the surface, and known input in terms of an earth model, the problem is virtually identical to the local earthquake problem except for scaling. In the case of the trapped miner, source depth may be known quite accurately and the earth model may also be known relatively well compared to the typical earthquake investigation.

The limited objectives of this Part are to evaluate the effects on location errors of such factors as model uncertainties, timing errors, array geometry, and different classes of models. To carry out such evaluations we have calculated standard error maps, contoured in the horizontal plane, for three classes of models, three array geometries, and various combinations of input parameter errors. The results should be interpreted not so much as absolute error predictions but as resolution maps showing the relative effects of various assumptions. Caution is required in interpretation because systematic bias in, for example, model assumptions with respect to the real earth, may produce systematic errors not accounted for by the error analysis. On the other hand, relative resolving power of the given configuration is properly indicated.

III. METHOD OF CALCULATION AND PRESENTATION

The method of error prediction is described by Peters and Crosson (1972). A velocity model is chosen from which pulse travel times can be calculated. The normal equations are formed for the non-linear least squares solution for the source at a given location, and the source location errors are calculated. The process is repeated for a number of locations in an x-y grid and the resultant values are contoured.

Statistical weighting is used so that data with large relative errors do not influence the calculations as strongly as data of higher accuracy. Thus, in the least squares method each station in the array is weighted with the reciprocal of the square of its associated uncertainty, which is a function of the errors in the model parameters and the derivatives of the travel time to that station with respect to these parameters. All velocity models used in these calculations are laterally homogeneous, velocity varying only with the z coordinate. The catalog of resulting contour maps numbered 1 through 24 is included in this Part. A standard format with four machine-plotted maps for each case is presented. The four plots are respectively σ_x, σ_y representing rms error in x and y coordinates, σ_z representing rms error in z when z is not fixed, and σ_{tot} representing the rms error in all three coordinates.

$$\sigma_{tot} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \qquad (1)^*$$

The σ's are to be understood as estimated standard errors or one standard deviation of a normal distribution, so that the probability is 68% that the estimated value lies within σ of the true value (and 95% that it lies within 2σ of the true value). The x coordinate is toward the top of each diagram and y is toward the right side. Where σ_z contours are not plotted, the depth was assumed known and fixed at 600 feet. All calculations are based on a source depth of 600 feet. A scale in feet is indicated on each diagram and all error values are in feet. Crosses mark 500 feet from the array center in both x and y for each diagram, and squares indicate station locations. Contours are labeled with their respective error values and it should be pointed out that contour intervals are variable to better illustrate a wide range of error characteristics. Thus, care must be exercised in directly

* References to Figures, Tables, and Equations apply to those in this Part unless otherwise noted.

comparing different diagrams. Contours are plotted in multiples of 10, 25 or 100 feet. Thus, for example, if the minimum error contour in a plot is 40 feet, the error never falls to 30 feet, but may be as low as 31 feet at some points.+ Each diagram is labeled as to the velocity model used and the errors incorporated into the calculations. Note that where a 5% model error is used it means that 5% error was assumed in <u>all</u> model parameters. For example, with a layered model both layer velocities and interface depths are included. For the linear velocity model both the surface velocity and the velocity gradient are included. An error of p% in the data means they are assumed to have been selected from a normal distribution about the true value which has a standard deviation of p%.

\+ The minimum total error is shown in each plot.

<u>Note</u>: Due to an error in scaling for the plotter, the x and y scales in Runs 1, 2, 5, 6, 8, 9 and 12 differ by a ratio of 5:4. This is of no significance in interpretation.

IV. DISCUSSION OF ERROR MAPS

Run 1 shows the error distribution for a constant velocity model with no model error and 1 millisecond time error. It is useful for comparisons with other cases utilizing the hexagonal array. Errors increase rapidly outside the array margin. Differences in x and y plots result from small differences in the symmetry about these two directions. The general features of Run 1 are found in all the hexagonal array analyses.

A. Time Errors

If there are no model errors, i.e., the model is known exactly, then the relative effect of arrival time errors is large. For example, a comparison of Run 9 with 5 millisecond time error and Run 1 with 1 millisecond time error shows, as would be expected, an increase in location error by a factor of 5. On the other hand, if model error is present, a change from 1 to 5 millisecond time error has a much smaller net effect, as illustrated by a comparison of Runs 10 and 3 for a linear velocity case. The conclusion to be drawn is that compared to probable model errors, a few milliseconds of arrival time error have a small effect.

However, once arrival time errors rise to 10 ms and above, they begin to dominate model errors. Location inaccuracies again rise roughly linearly with arrival time errors once these have risen to 15-20 ms or so (Runs 21-23).

B. Model Errors

Model errors exert strong control on the resolution capability of a given configuration. Comparison of Runs 1 and 12, where the only differences are in error assigned to the constant velocity model, illustrates this feature. Similarly, a comparison of Runs 2 and 3 illustrates the same effect for the linear velocity model. The total error almost quadruples at the array margin when going from no error to 5% model error.

C. Model Differences

The differences in error structure as a function of changing models are not large in most cases. Generally, models with velocity increasing with depth, such as a linear velocity or layered model, offer superior resolution compared with a constant velocity halfspace, especially with respect to depth. Since the real earth is perhaps closer to the linear or layered models, some advantage is gained. The total error diagrams for Runs 1 and 2 illustrate the model dependent effect between the constant-velocity and linear-increase models, when the models are assumed to be exact. Note that the linear velocity increase model yields better resolution within the array proper than the uniform halfspace model; however the rate of deterioration of location accuracy outside the array is more rapid than for the halfspace. Examination of the layered model results of Run 14 shows behavior similar to that of the linear model.

D. Depth Known

Several cases were calculated to show the resultant increase in resolution when it is assumed the depth is known, as it could well be in the case of trapped miners known to be at specific levels. Run 4 compared with Run 3 shows the fairly marked effect of fixing depth for two otherwise identical cases. Resolution within the boundaries of the array becomes very uniform. Comparison of σ_{tot} for these two cases is not really meaningful since σ_{tot} for the z unknown case is dominated by σ_z. The same kind of improvement is noted for all fixed vs. free depth comparisons such as Runs 14 and 16, and Runs 7 and 8. However, this result, as discussed in Section IV-G, appears to be invalid if a homogeneous earth model is used, when a variable depth allows a better lateral location accuracy to be achieved.

E. Changing Array Dimensions

Previous resolution studies suggest that improved control may be obtained if an array does not have a high degree of symmetry. The explanation for this phenomenon is that arrival time data from a highly symmetrical array may be largely redundant and thus lacking in location "information". Less symmetrical configurations are illustrated in Runs 11 and 13, both for a linear-velocity model, and in Run 15 for a layered-model. In Run 11, a modified "H" array shows significant improvement over the highly symmetric hexagonal array used in Run 3. Similarly, the "stretched" hexagonal array used for Run 13 shows slight improvement over the hexagonal array of Run 3. Thus array configuration is an important factor in the design of the system.

F. Layered Models

Runs 14 through 19 are comparative cases run on both 2- and 4-layer models. The results are not dramatically different from similar cases run with the linear model which was chosen as a reasonable representation of the 4-layer model. Since, for all cases, the source lies in the deepest layer, the improved depth control effects noted in earthquake studies when refractions occur (Peters and Crosson, 1972) are not observed here. However, in contrast to the linear model, model errors varying as a function of depth could be represented effectively in a layered model.

G. Calibration of Earth Models

A relative calibration of earth models is exhibited in the results of computed locations shown in Table 2. Arrival times from seismic events at a fixed depth of 600 feet but varying lateral positions relative to the hexagonal array were generated using a 4-layer model as follows:

TABLE 1

4-LAYER MODEL

Depth	P-Wave Velocity
0 ↓ 10	2,000 fps
10 ↓ 100	4,000
100 ↓ 300	8,000
300	
half-space	12,000

The locations for these events were then computed using two simpler "best fit" models.

(i) Homogeneous half-space, $V_p = 8{,}500$ fps

(ii) Linear velocity model, $V_p = 4{,}200 + 500z$ fps

The term "best fit" in this context means that these models best fitted a travel time curve for the 4-layer model in a least-squares sense over distances of interest to this experiment.

It can be seen from Table 2 that the linear velocity model provides an excellent fit to the "real" 4-layer earth both inside and outside the hexagonal array; the homogeneous half-space is a much less satisfactory approximation for location purposes, deteriorating particularly rapidly at the boundaries of the array. Interestingly, the homogeneous half-space model always provides a more accurate lateral fix when the depth of the seismic event is allowed to vary from its true value rather than when it is fixed. This is only true for the linear velocity model when the seismic event falls within the array. When the depth is allowed to vary, there is a corresponding inability on the part of the approximate models to match the true time of occurrence of the seismic event, as shown in Table 2.

Preliminary conclusions that may be drawn from these results are that in practical terms a linear velocity earth model, which is computationally much easier to handle, may be used to represent a layered earth for location purposes without introducing serious errors; secondly, if a homogeneous earth model is used, it may be wiser to let the depth vary even if it is known, since errors in arrival times will predominantly introduce an error in the computed z coordinate which, if not left free to "compensate" for this, will cause larger errors in the x and y coordinates.(As the seismic event moves away from the center of the array, arrival times become more sensitive to the x and y coordinates; hence, this reasoning eventually breaks down, as shown by the results obtained for the linear velocity model.)

V. REFERENCES

Peters, D.C. and Crosson, R.S. (1972). Application of Prediction Analysis to Hypocenter Determination Using a Local Array, Bull. Seis. Soc. Am., v. 62., pp 775-788

Wolberg, J.R. (1967). Prediction Analysis, D. Van Nostrand Co., Inc., Princeton, N. J.

TABLE 2

COMPARISON OF LOCATION ERRORS FOR TWO APPROXIMATE MODELS

Arrival Times Generated Using 4-Layer Earth Model-Event Locations Computed Using "Best Fit" Constant Velocity Half-Space and Linear Velocity Models with Assumed Model Errors of 5%

Location Array: 7 Seismometer
Hexagonal Array: 600 ft. side

Actual Event Location (feet)		Computed Event Location (feet) Const. Vel. Depth Varied	Const. Vel. Depth Fixed	Lin. Vel. Depth Varied	Lin. Vel. Depth Fixed
x	100	92.8	86.9	100.5	97.8
y	0	0	0	0	0
z	600	694.9	600	632.3	600
t	0 secs	-0.009	0	-0.001	0
x	300	291.8	258.2	301.1	291.6
y	0	0	0	0	0
z	600	706.6	600	629.1	600
t	0 secs	-0.015	0	-0.001	0
x	500	523.5	423.4	495.3	485.6
y	0	0	0	0	0
z	600	717.7	600	613.6	600
t	0 secs	-0.026	0	0	0
x	700	758.2	599.0	666.7	683.8
y	0	0	0	0	0
z	600	724.6	600	587.3	600
t	0 secs	-0.036	0	0	0
x	900	955.3	781.2	818.0	913.4
y	0	0	0	0	0
z	600	729.1	600	554.3	600
t	0 secs	-0.042	0	0.006	0

TABLE 3

SUMMARY OF ERROR DIAGRAMS

Run #	Array Type	Station Spacing, ft.	Velocity Model	Parameter Error σ_v(%)	σ_t(sec.)	Depth Fixed?
1	Hex	600	Con	0	.001	
2	Hex	600	Lin	0	.001	
3	Hex	600	Lin	5%	.001	
4	Hex	600	Lin	5%	.001	*
5	Hex	1200	Con	0	.001	
6	Hex	1200	Lin	0	.001	
7	Hex	1200	Lin	5%	.001	
8	Hex	1200	Lin	5%	.001	*
9	Hex	600	Con	0	.005	
10	Hex	600	Lin	5%	.005	
11	H	600	Lin	5%	.001	
12	Hex	600	Con	5%	.001	
13	Mod Hex	450	Lin	5%	.001	
14	Hex	600	2 Lay	5%	.001	
15	H	600	2 Lay	5%	.001	
16	Hex	600	2 Lay	5%	.001	*
17	Hex	600	2 Lay	5%	.005	*
18	Hex	600	4 Lay	5%	.001	*
19	Hex	600	4 Lay	5%	.005	*
20	H	600	Lin	5%	.001	*
21	Hex	600	Lin	5%	.010	*
22	Hex	600	Lin	5%	.015	*
23	Hex	600	Lin	5%	.020	*
24	Hex	600	Lin	1%	.005	*

* indicates depth fixed for error computations.

TABLE 3 (Continued)

Con -- Constant Velocity Model: V_p = 12,000 fps

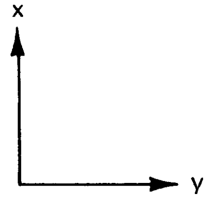
Lin -- Linear Velocity Model: V_p = 5,000 + 18 fps

2 Lay -- Two-Layer Model:

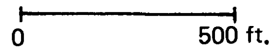
Depth, ft.	Velocity, fps
0-200	6,000
200 +	10,000

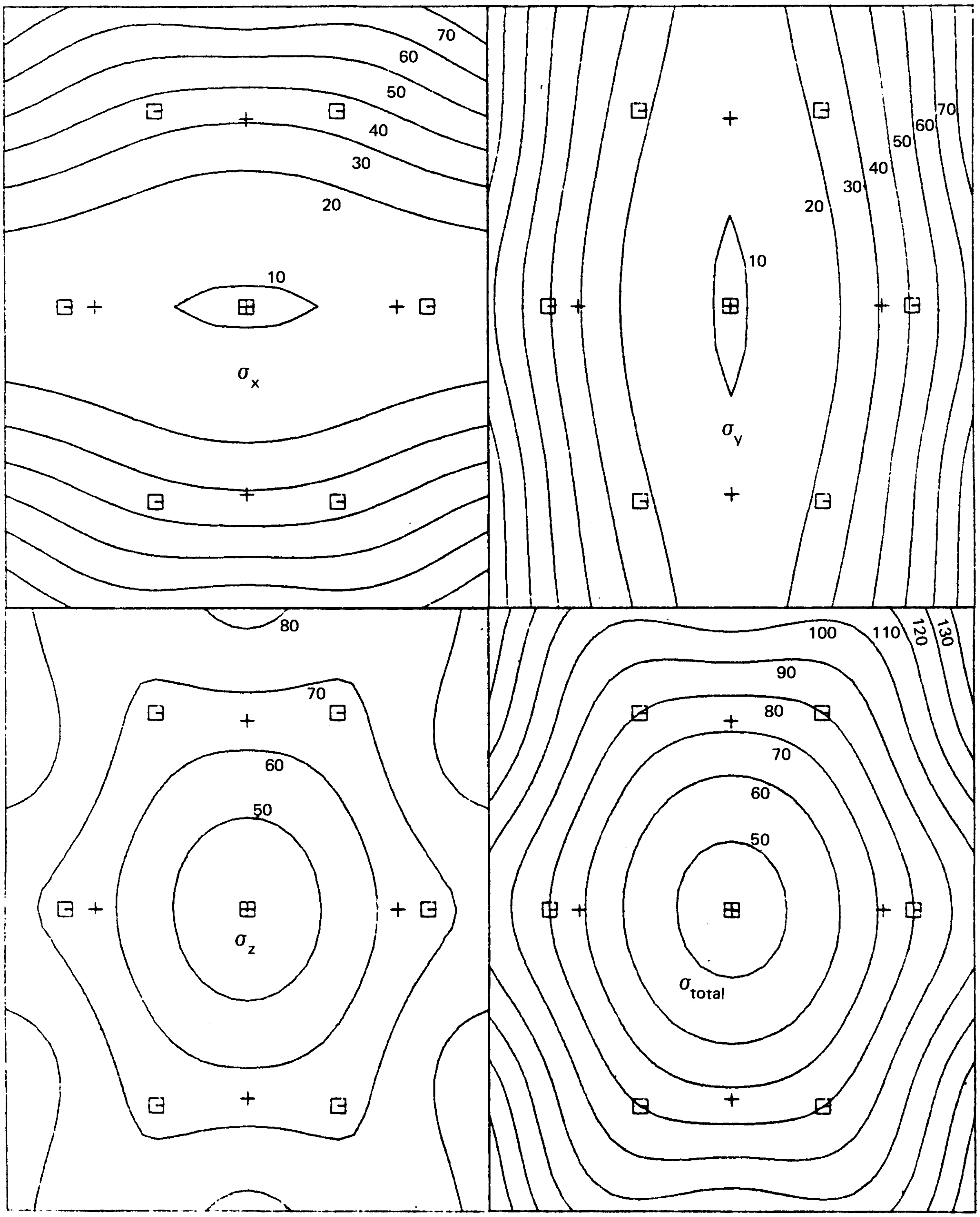
4 Lay -- Four-Layer Model:

Depth, ft.	Velocity, fps
0-10	2,000
10-100	4,000
100-300	8,000
300 +	12,000



NOTE: Vertical and Horizontal
Scales are Different
in the Ratio of 5:4
on Some Runs





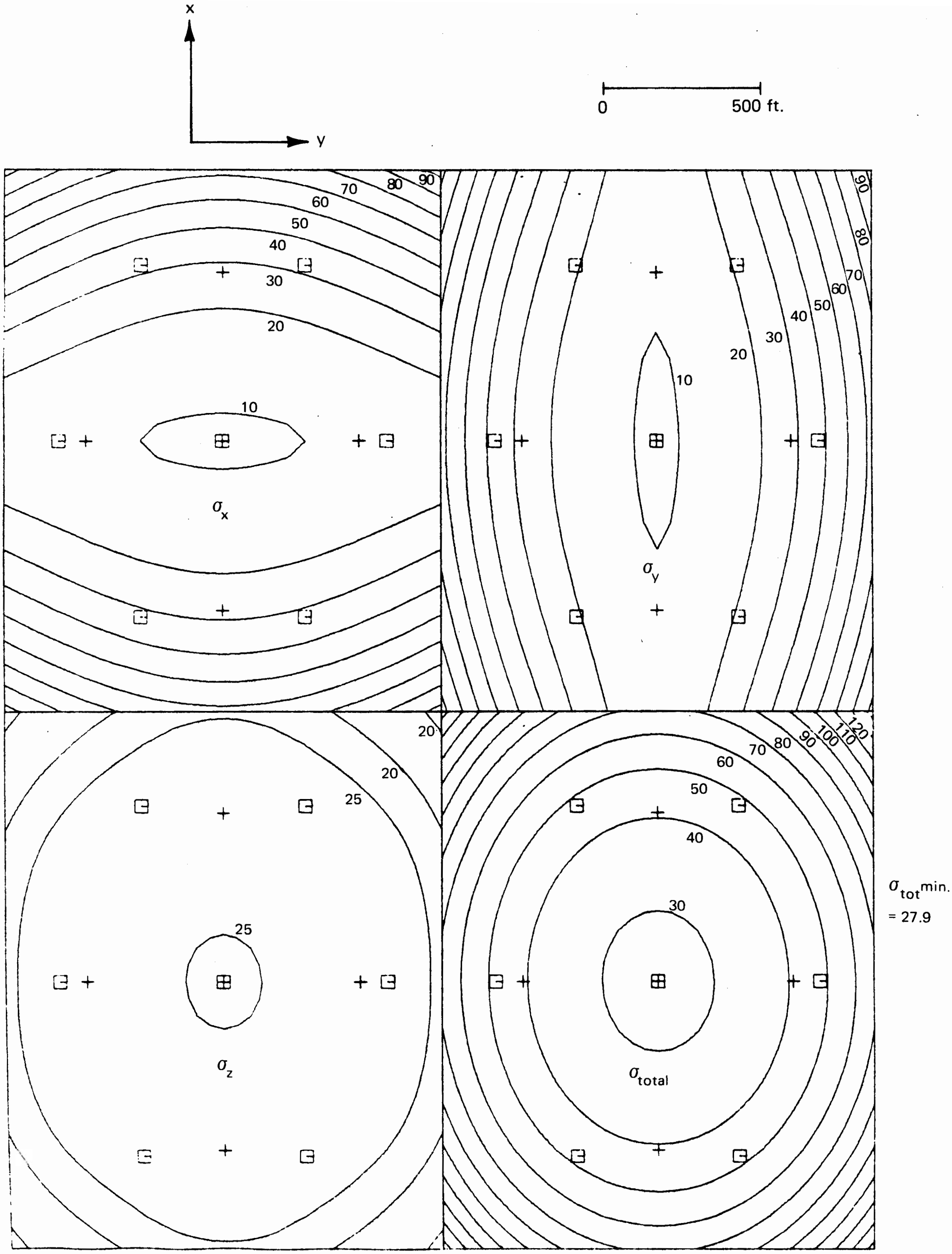
σ_{tot}min.
= 46.4

MODEL: Constant Velocity
$v = 12000$ fps
$\sigma_v = 0\%$
$\sigma_t = 1$ ms

Note Large Effect of
Time Error when
Model is Known

RUN 1

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

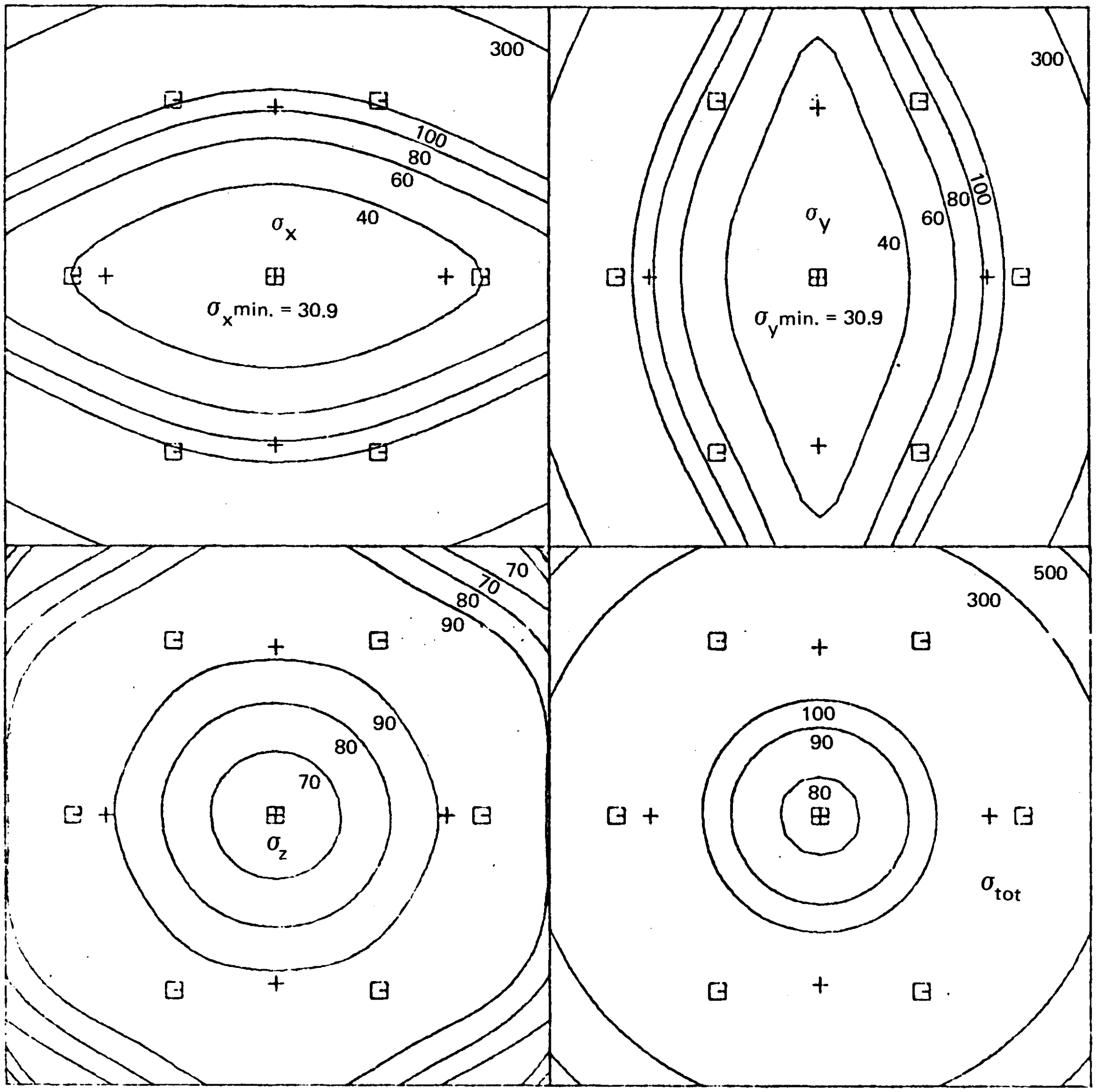


MODEL: Linear Velocity
v = 5000 + 18z
$\sigma_v = 0$
$\sigma_t = .001$

Linear Velocity
Gives Much Better
Depth Control

RUN 2

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

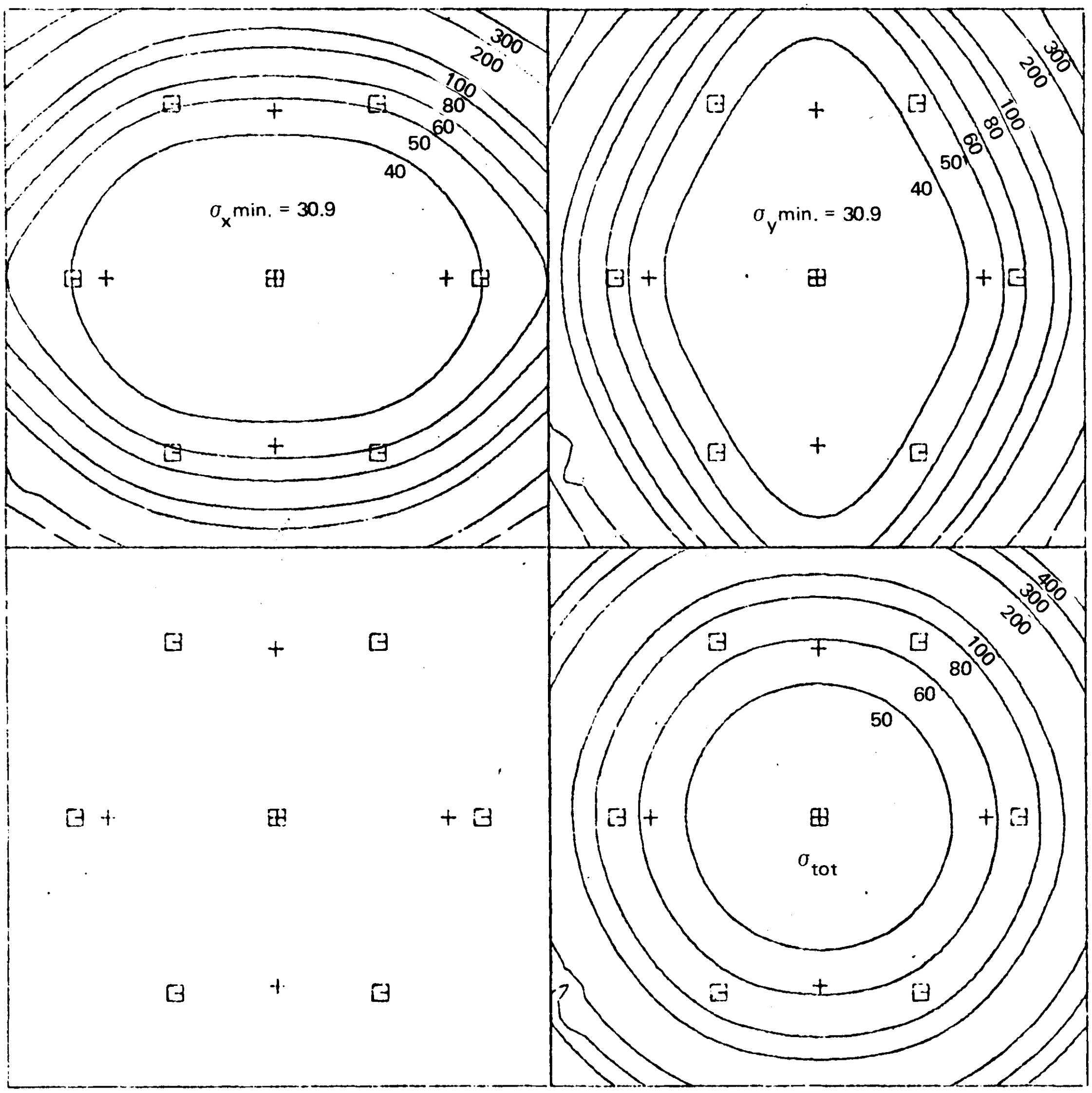


$v = 5000 + 18z$
$\sigma_v = 5\%$
$\sigma_t = .001$

σ_{tot}min. = 77.6

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 3

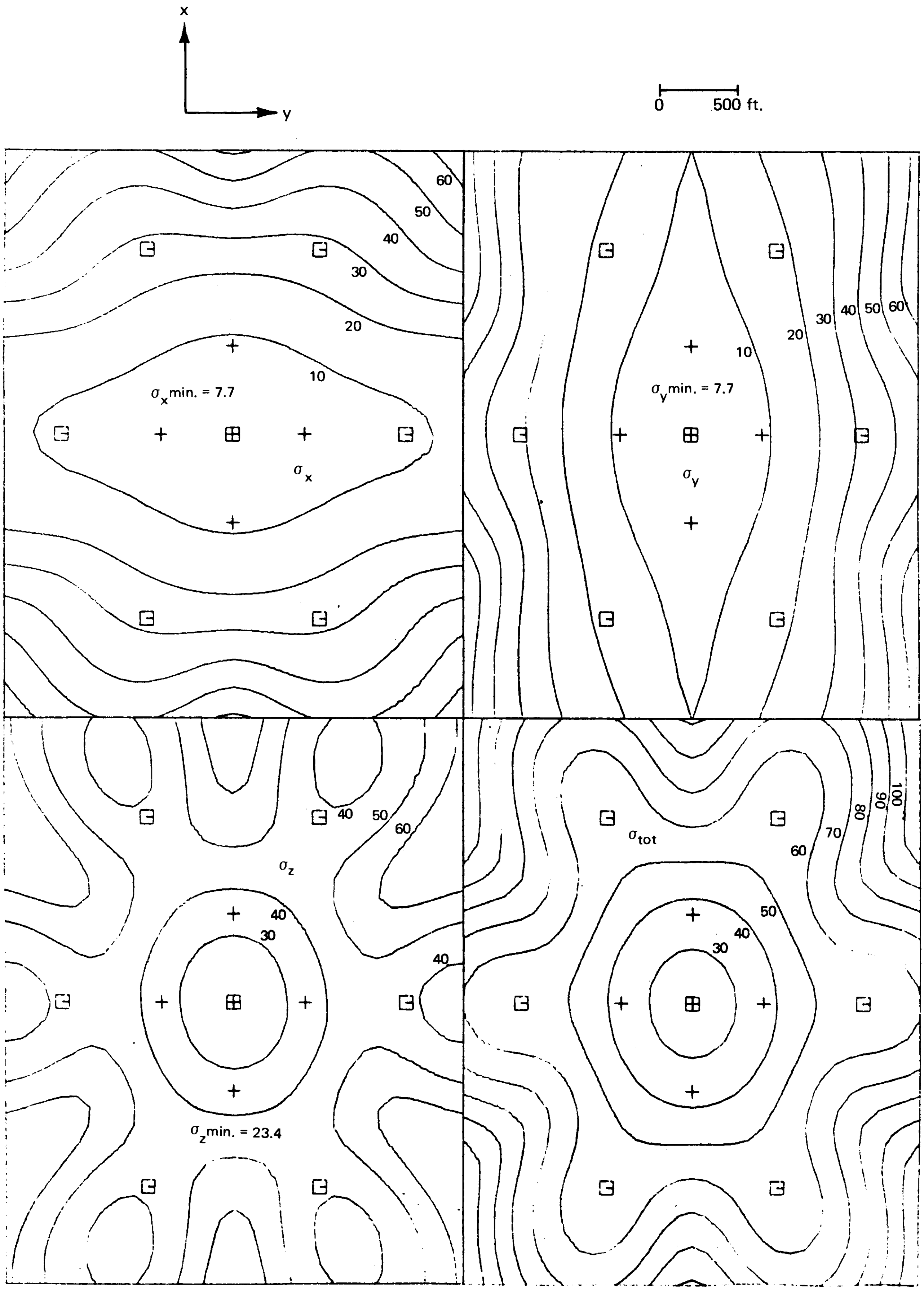


$v = 5000 + 18z$
$\sigma_v = 5\%$
$\sigma_t = .001$

σ_{tot}min. = 43.7

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

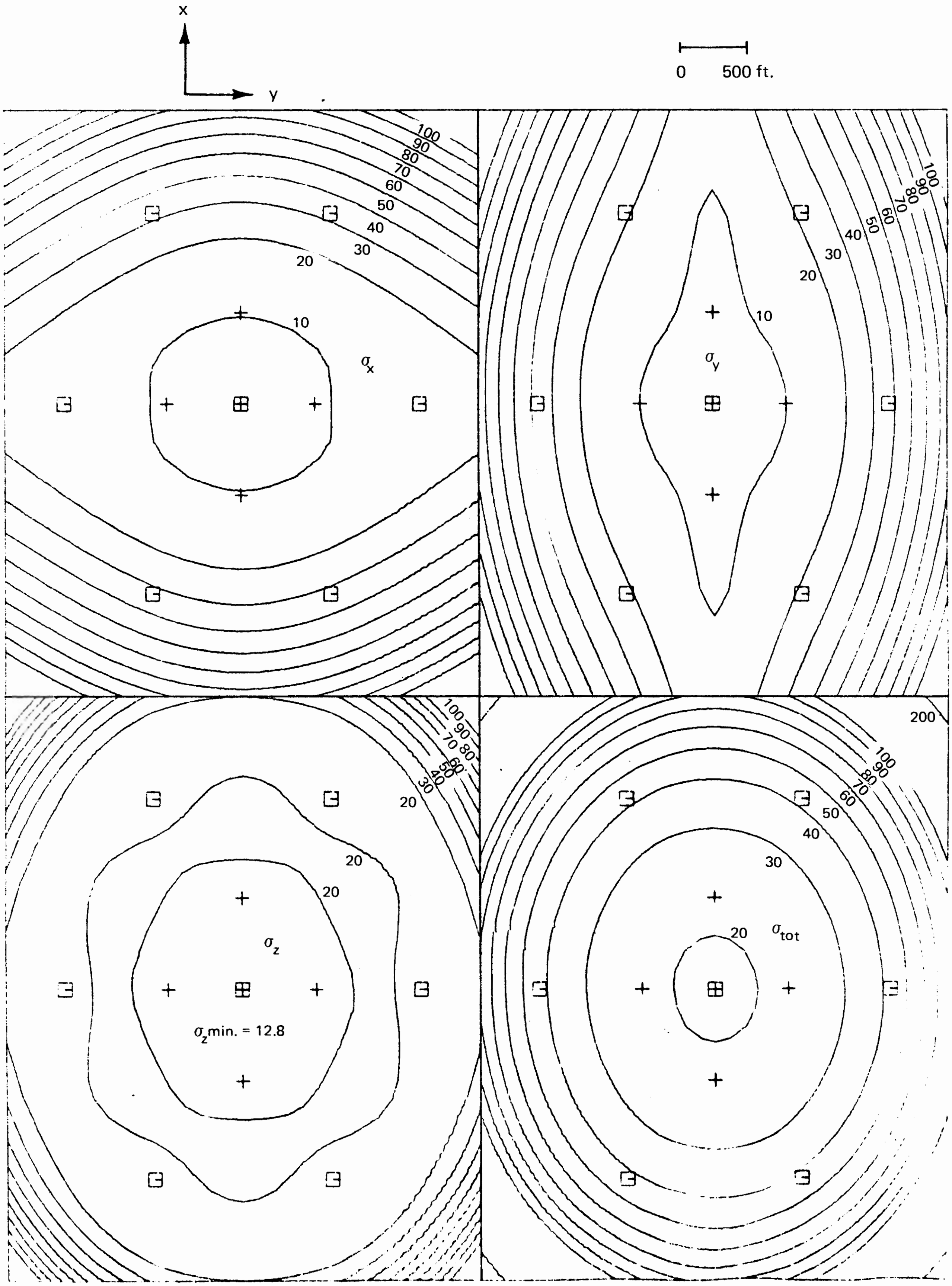
RUN 4



v = 12000
σ_v = 0
σ_t = .001
ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

Expanded array
Compare to 1; σ_{tot}min. = 25.9

RUN 5

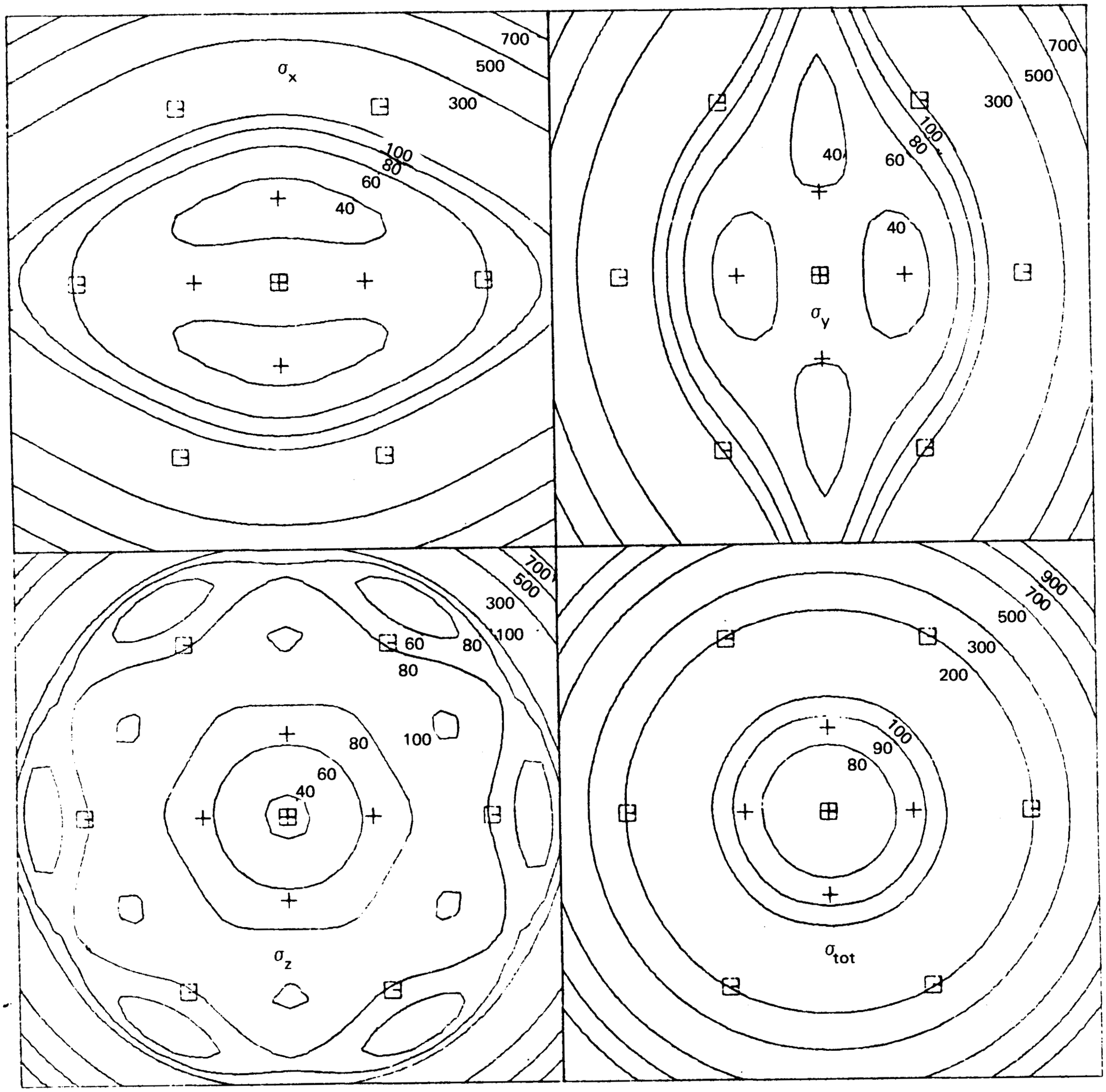


v = 5000 + 18z
$\sigma_v = 0$
σ_t .001

Expanded Array
Compare to 2; σ_{tot}min. = 18.7

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 6

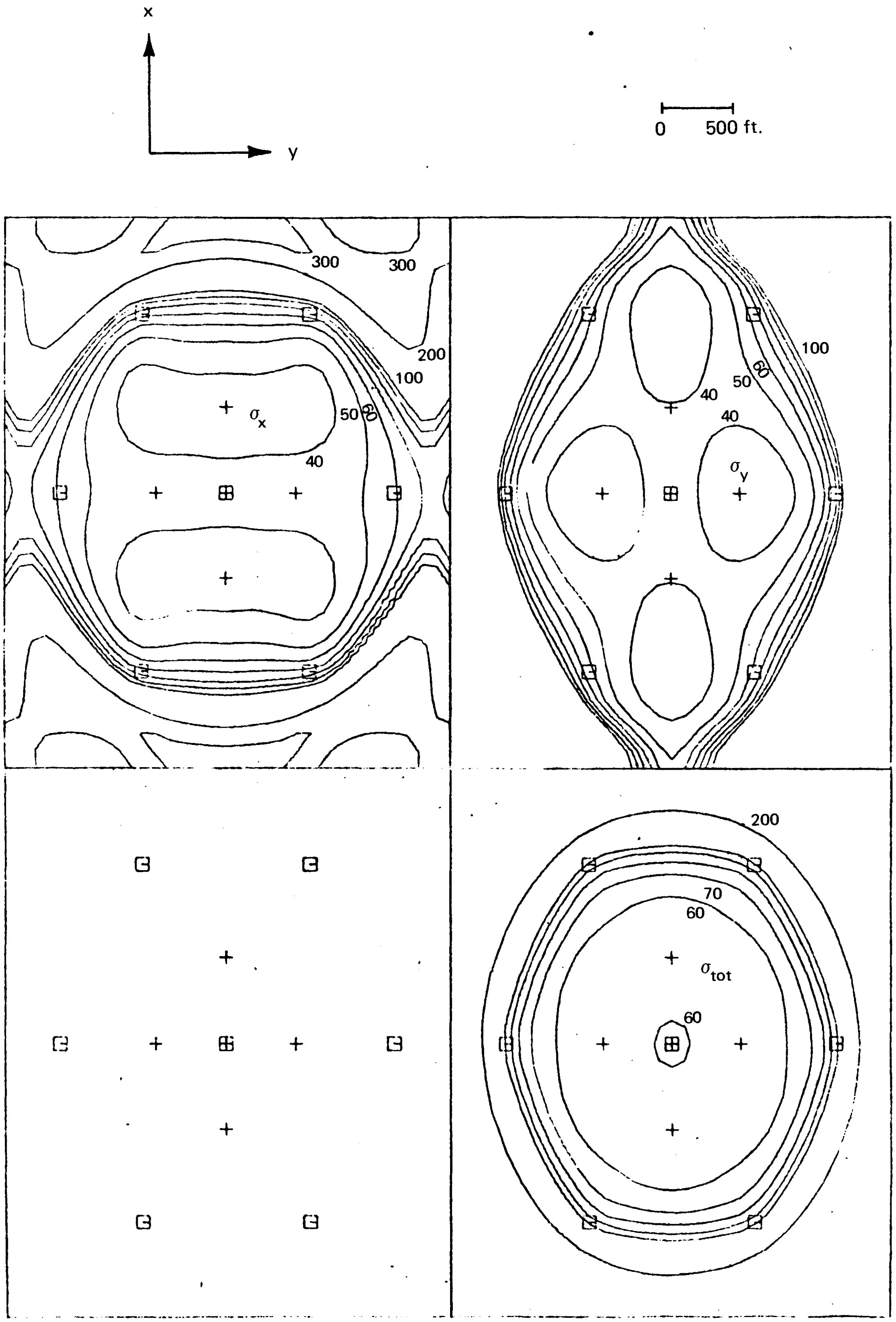


v = 5000 + 18z
σ_v = 5%
σ_t = .001
Double Array Dimensions

σ_{tot}min. = 71.7

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 7

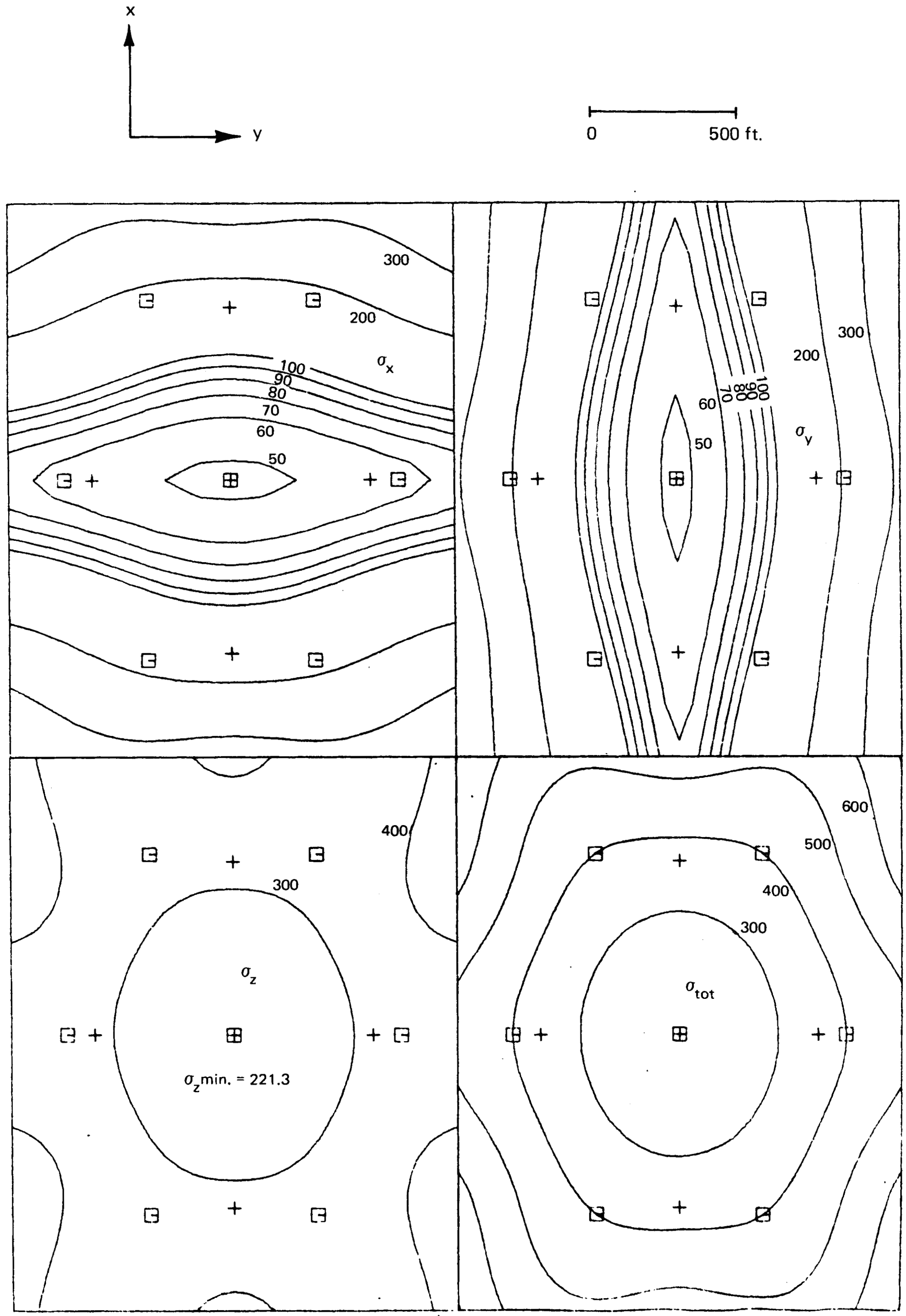


$v = 5000 + 18z$
$\sigma_v = 5\%$
$\sigma_t = .001$
Depth Fixed

Expanded Array
Compare to 4; σ_{tot}min. = 54.0

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 8

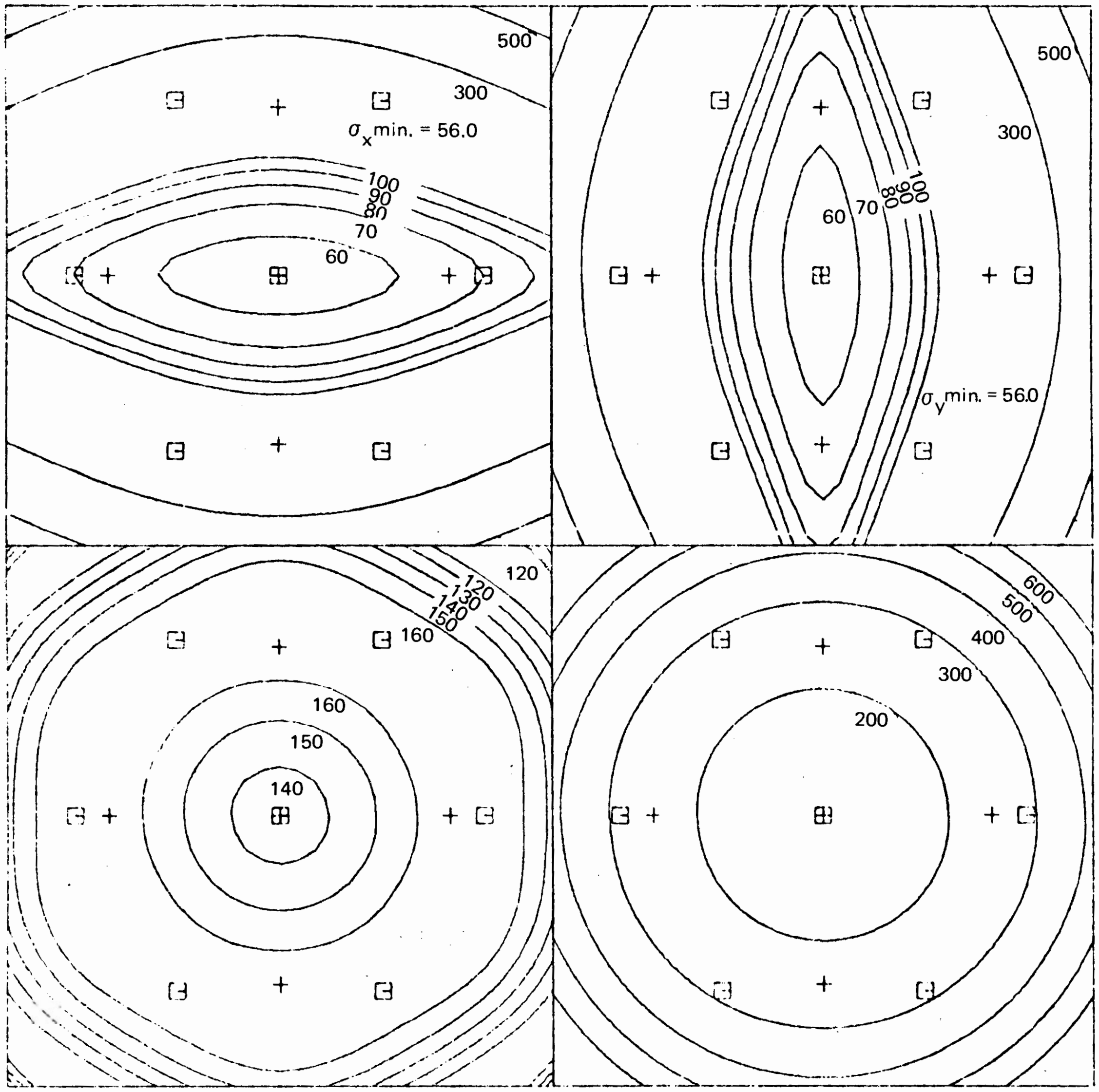


v = 12000
σ_v = 0
σ_t = .005

$\sigma_{x_i} = 5 \times \sigma_{x_i}$ (plot 1)
σ_{tot}min. = 231.9

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 9

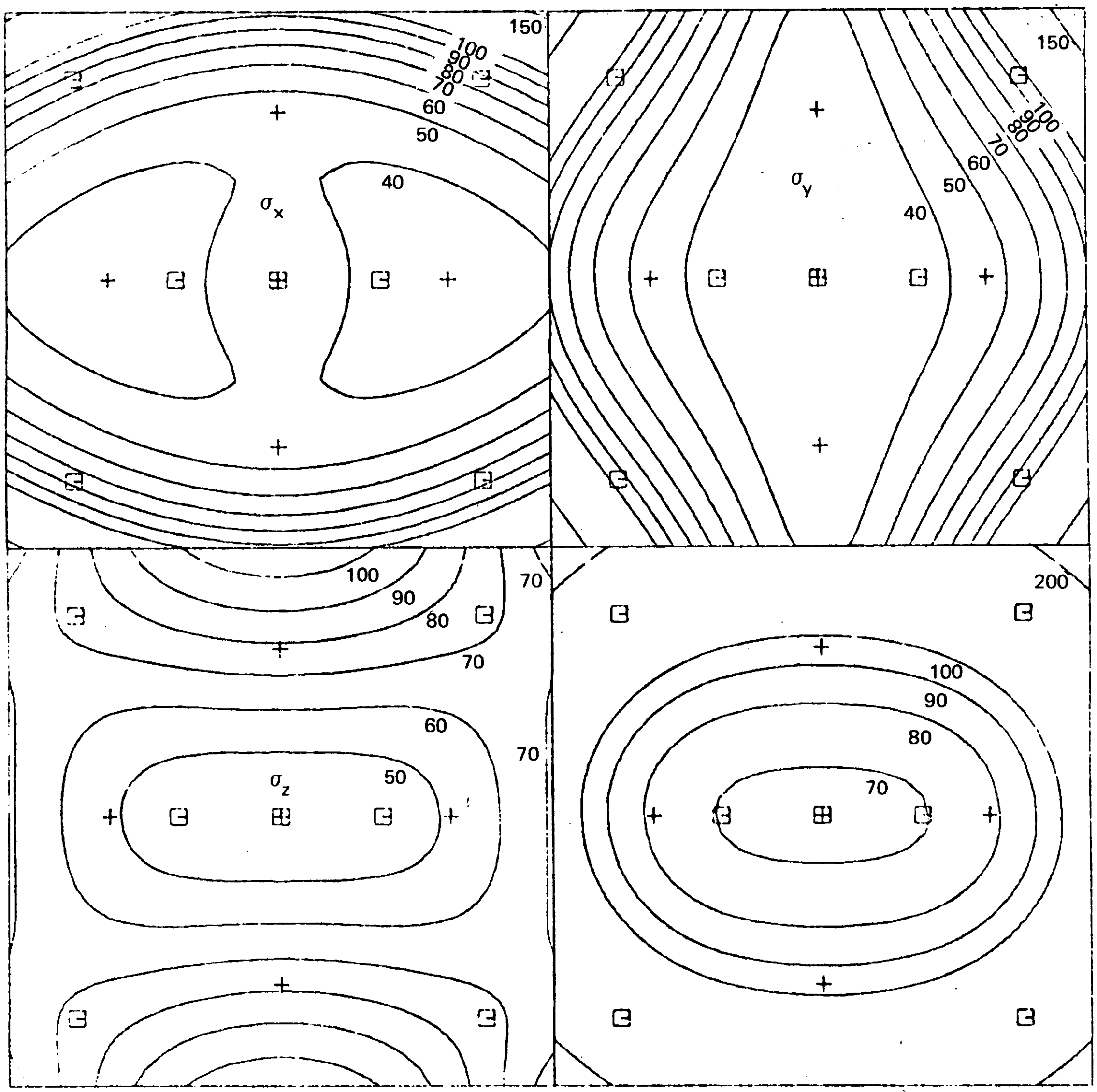


v = 5000 + 18z
σ_v = 5%
σ_t = .005

σ_{tot}min. = 156.0

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 10

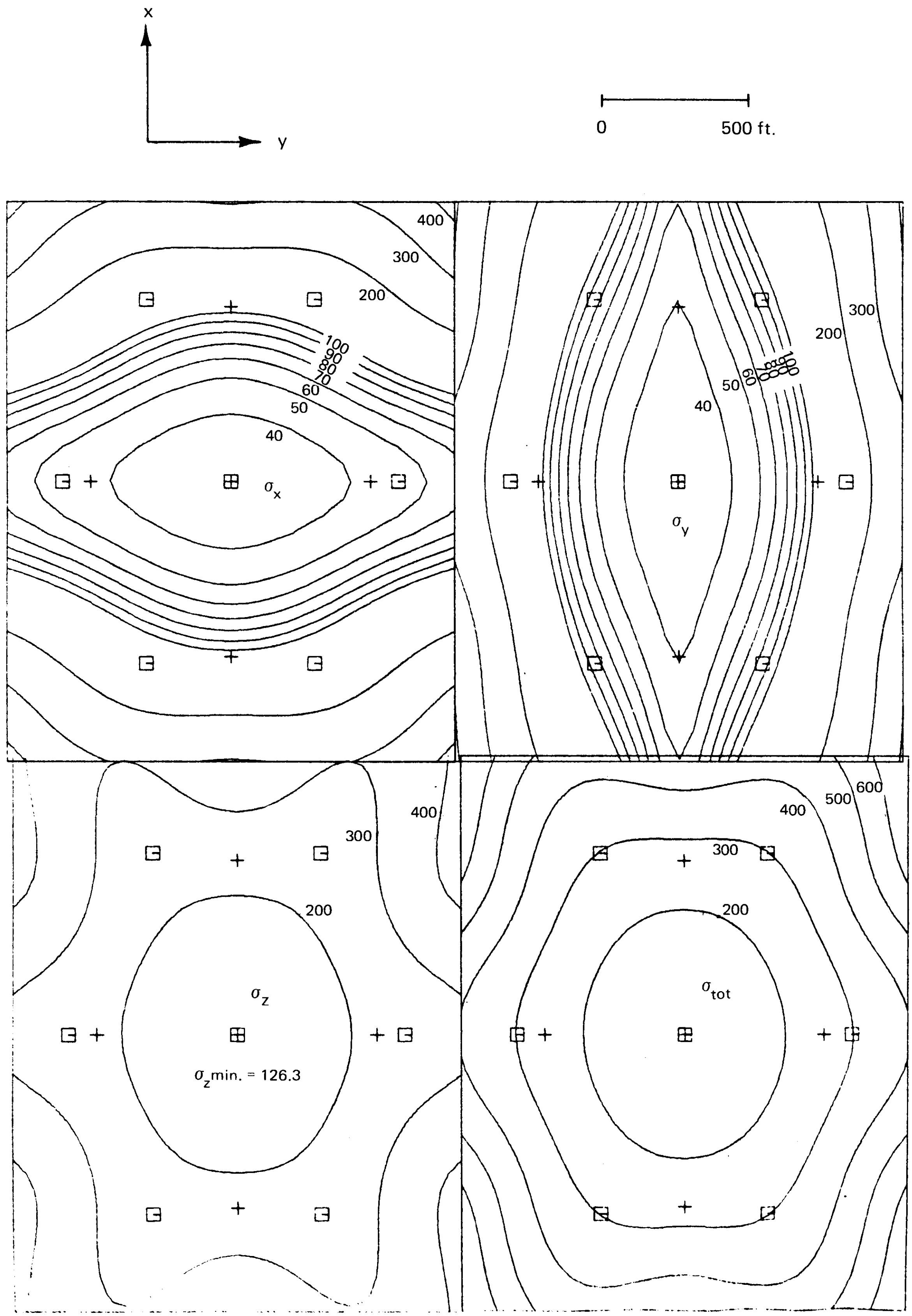


$v = 5000 + 18z$
$\sigma_v = 5\%$
$\sigma_t = .001$

σ_{tot}min. = 67.7

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 11

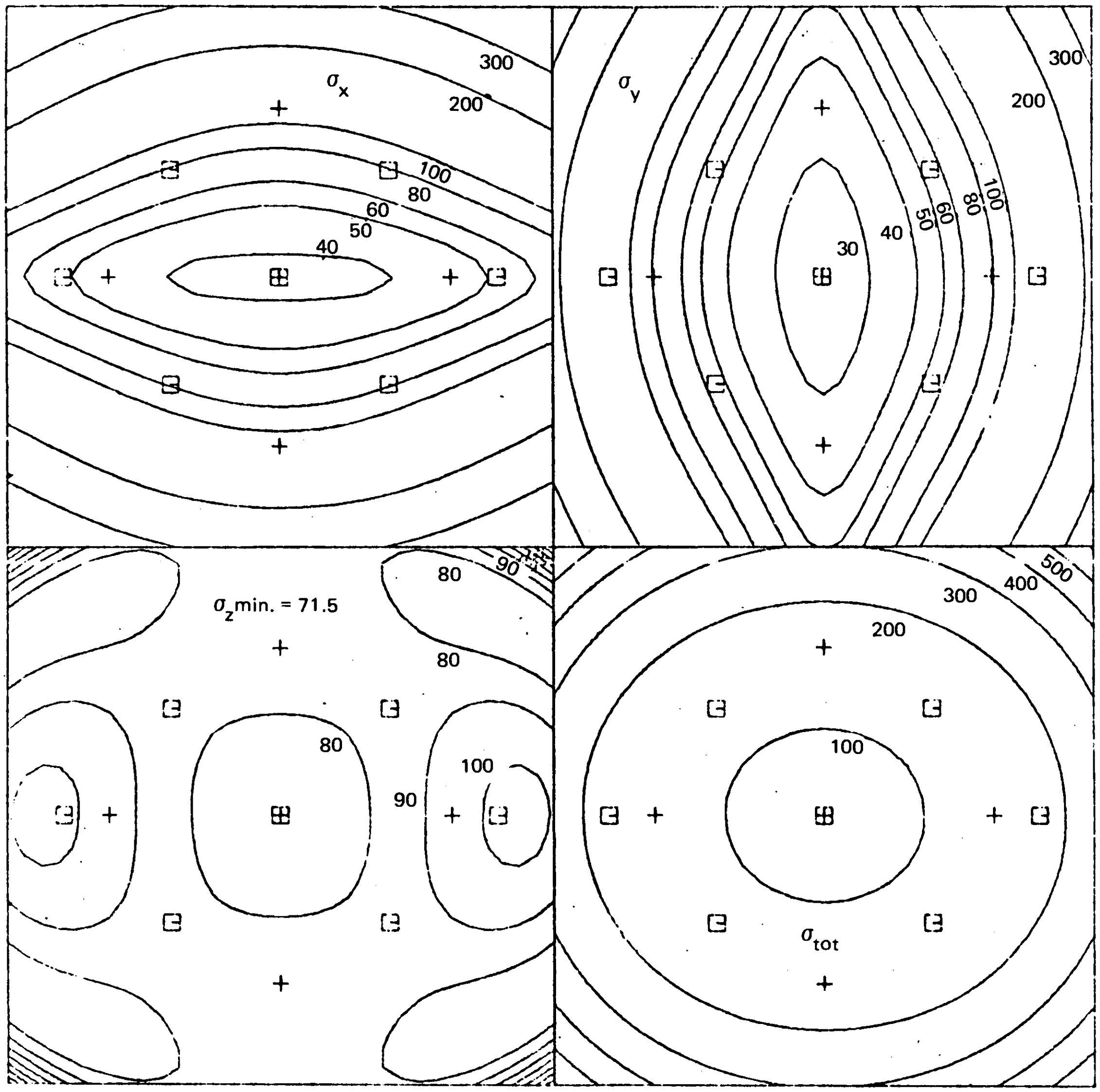


v = 12000
σ_v = 5%
σ_t = .001

Compare to 1, 3, 9
σ_{tot}min. = 136.2

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 12

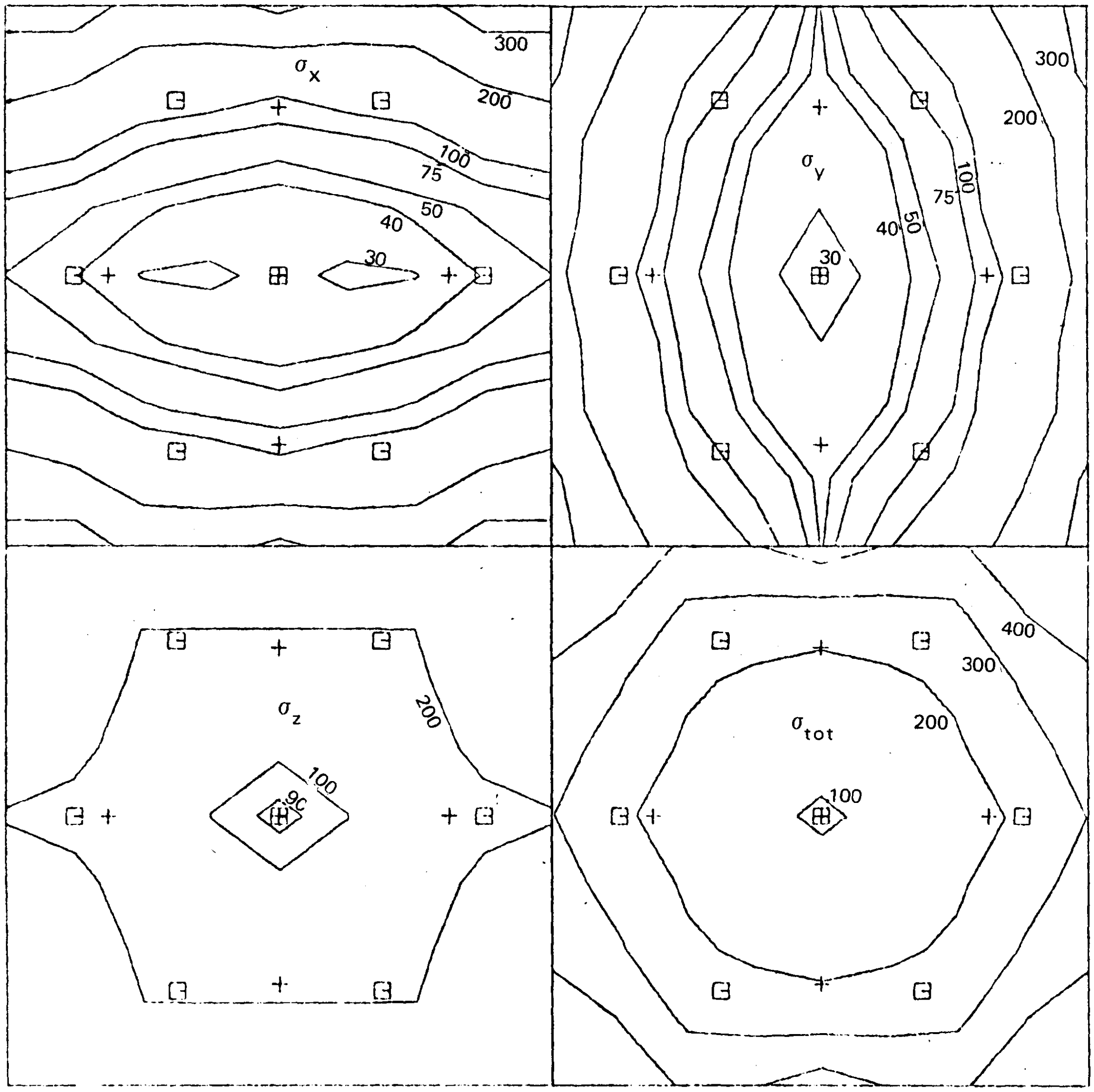


v = 5000 + 18z
σ_v = 5%
σ_t = .001

σ_{tot}min. = 85.8

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 13

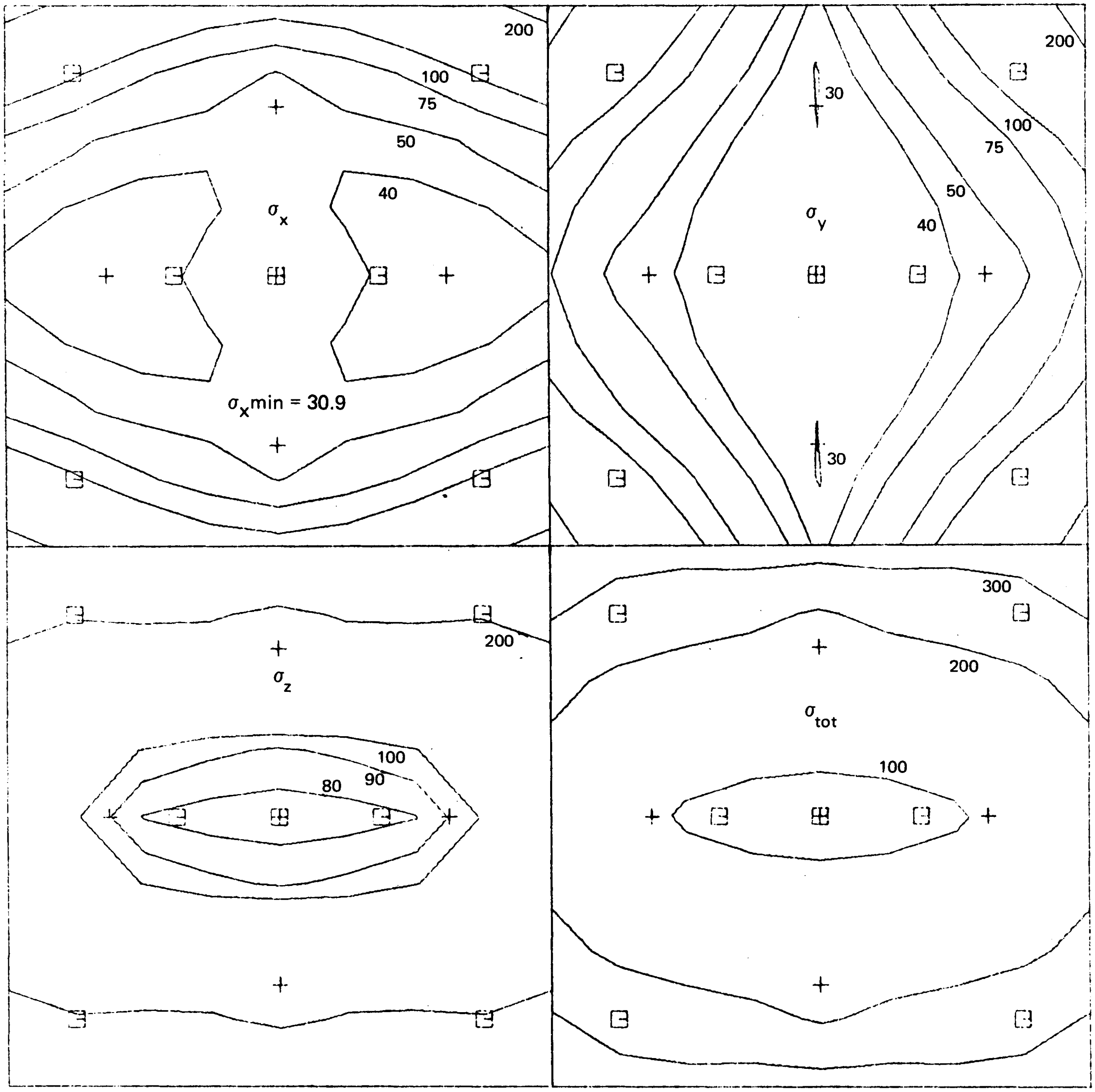


2 Layer Model
$\sigma_v = 5\%$
$\sigma_t = .001$

σ_{tot}min. = 95.2

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 14

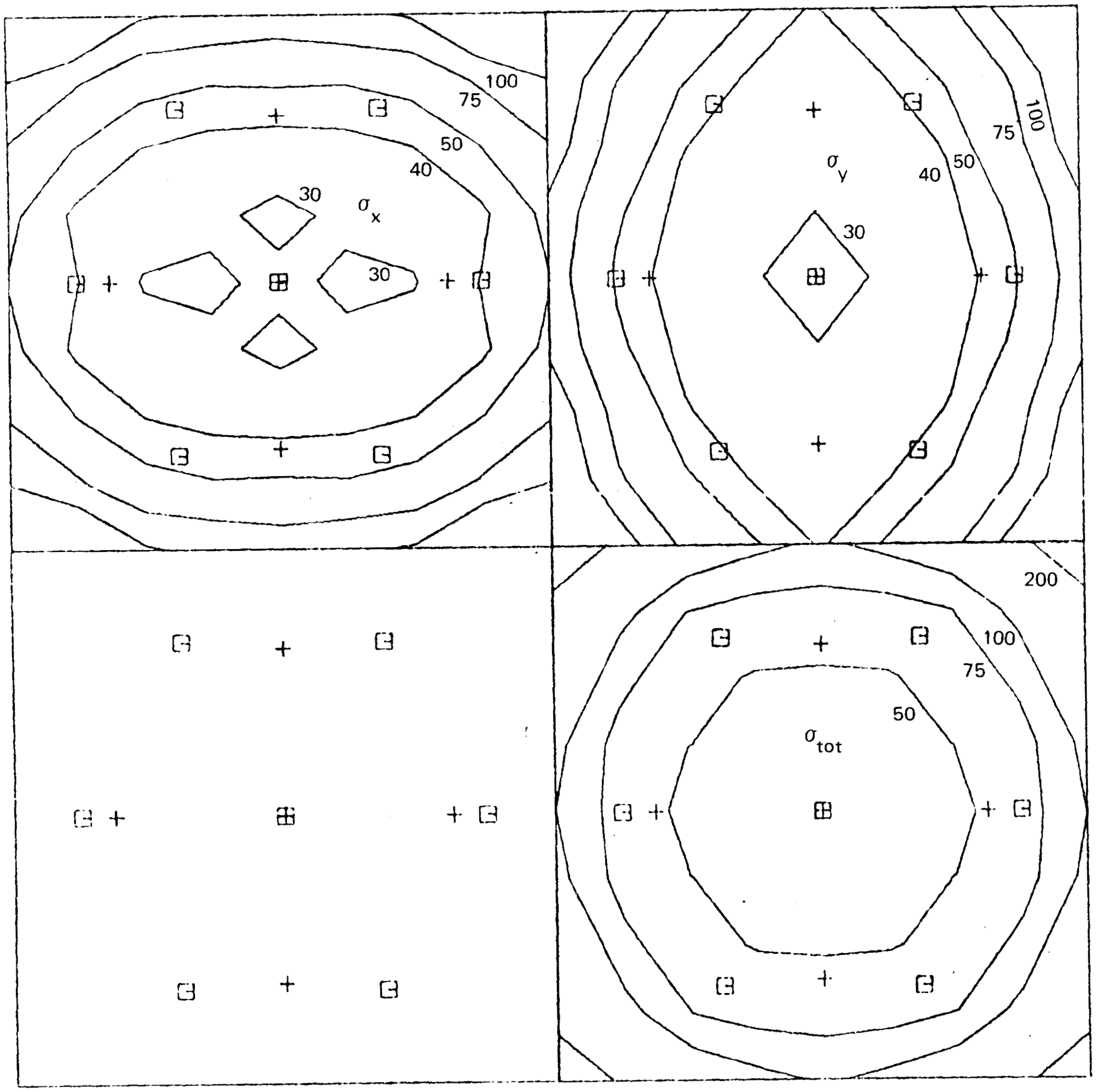


2 Layer Model
$\sigma_v = 5\%$
$\sigma_t = .001$

$\sigma_{tot} min. = 93.4$

ERROR CONTOURS IN FEET
SOURCE DEPTH — 600 FEET

RUN 15

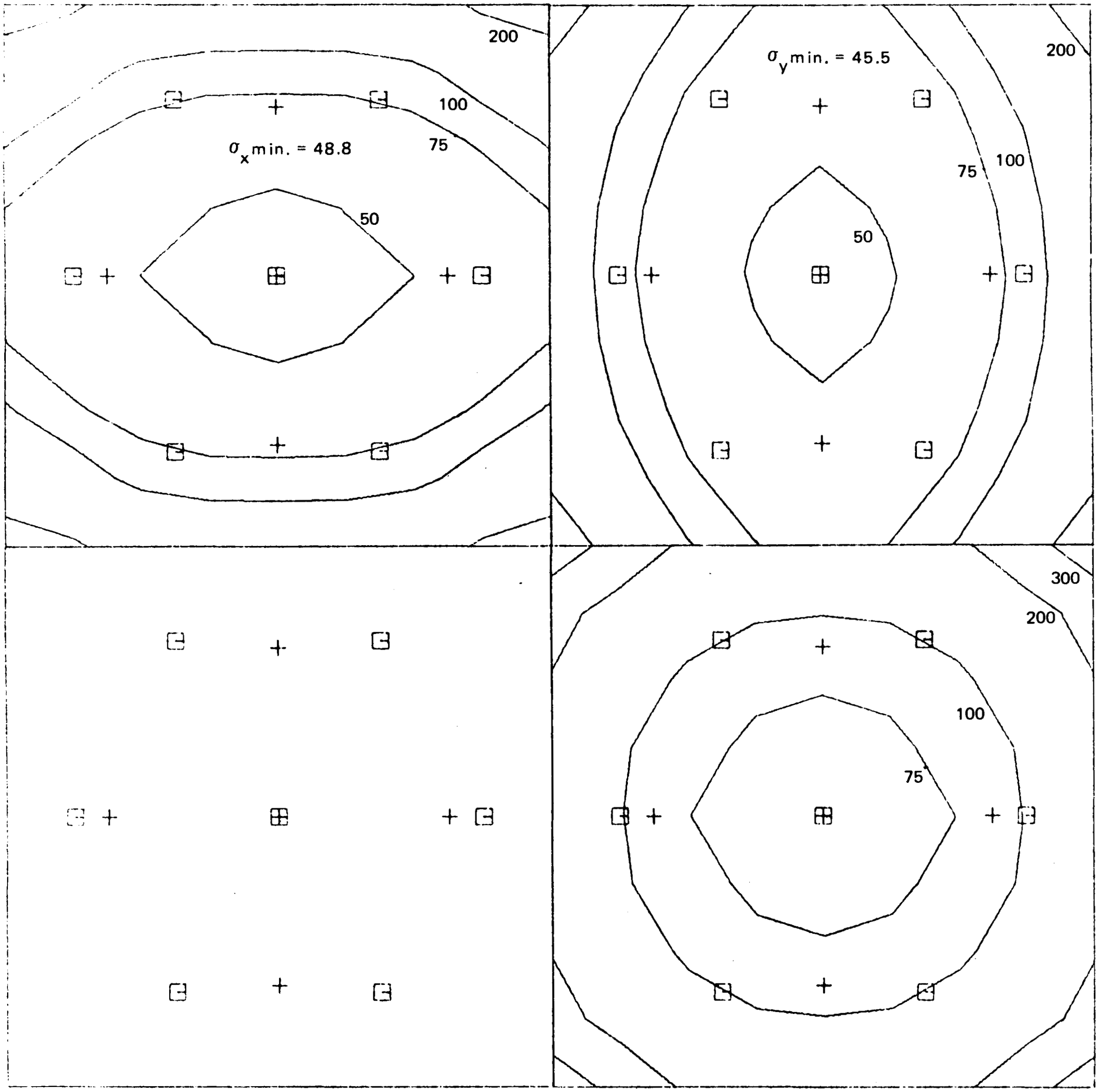


2 Layer Model
σ_v = 5%
σ_t = .001

σ_{tot}min. = 41.8

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 16

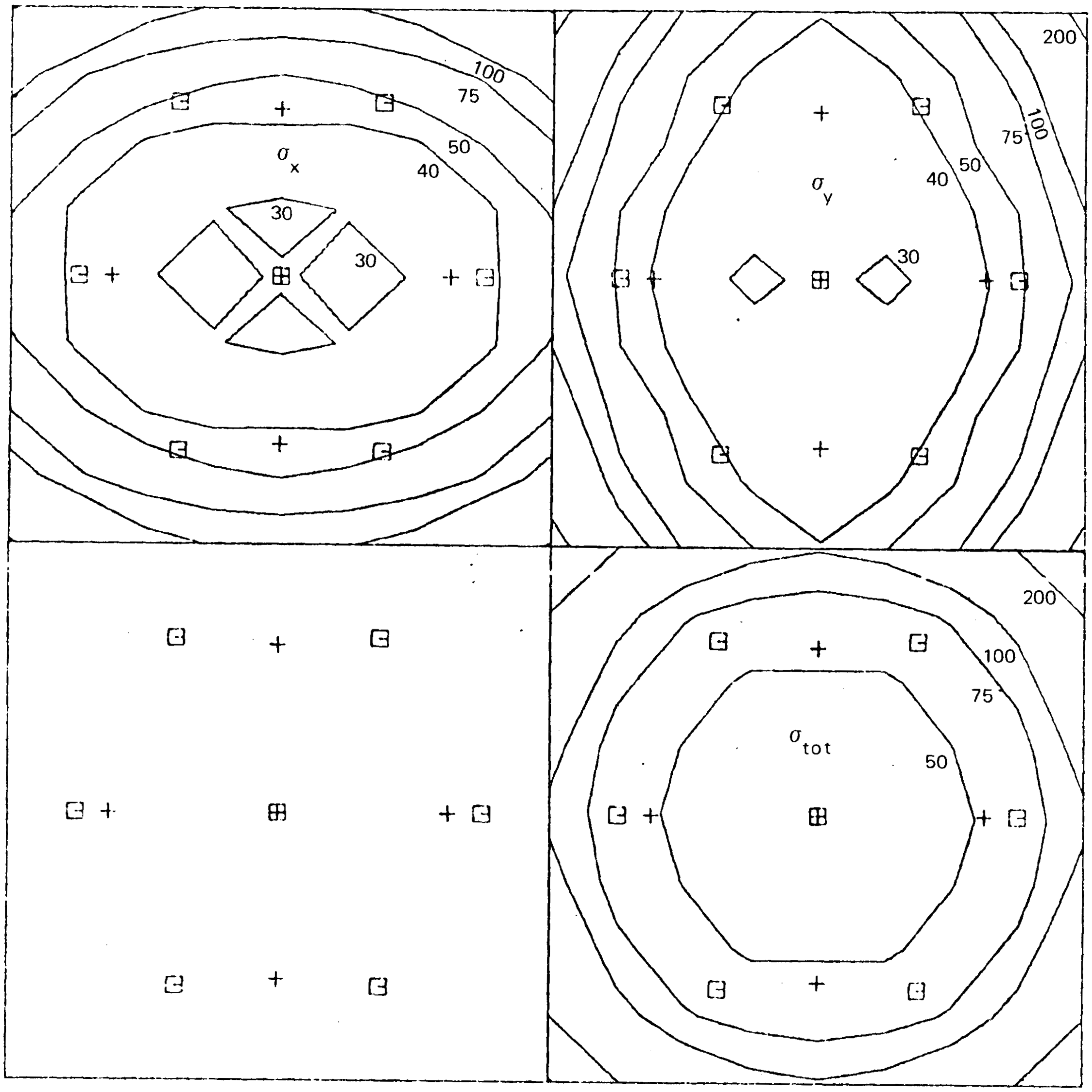


2 Layer Model
σ_v = 5%
σ_t = .005

σ_{tot}min. = 66.7

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 17

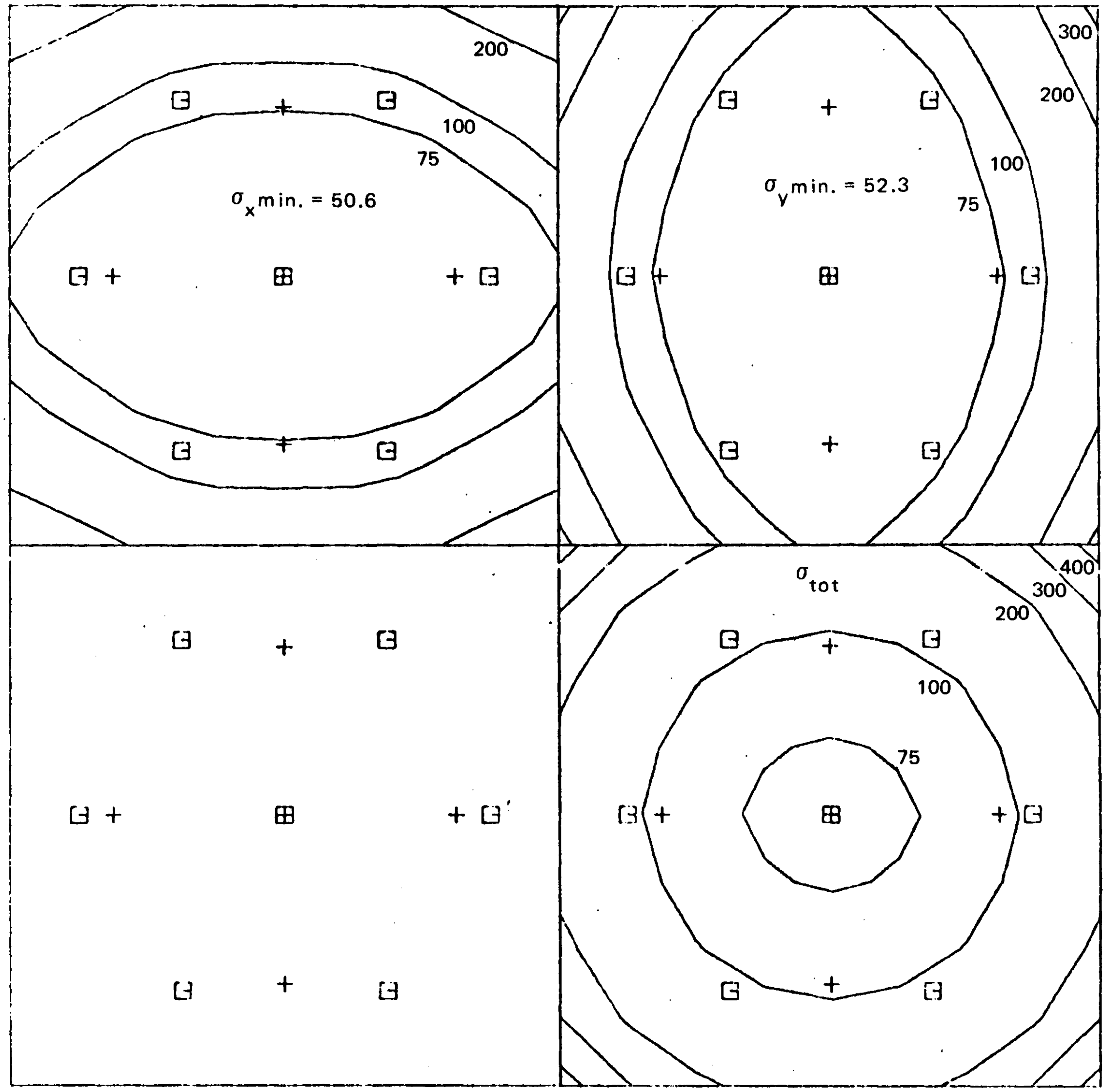


4 Layer Model
σ_v = 5%
σ_t = .001

σ_{tot}min. = 41.0

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 18

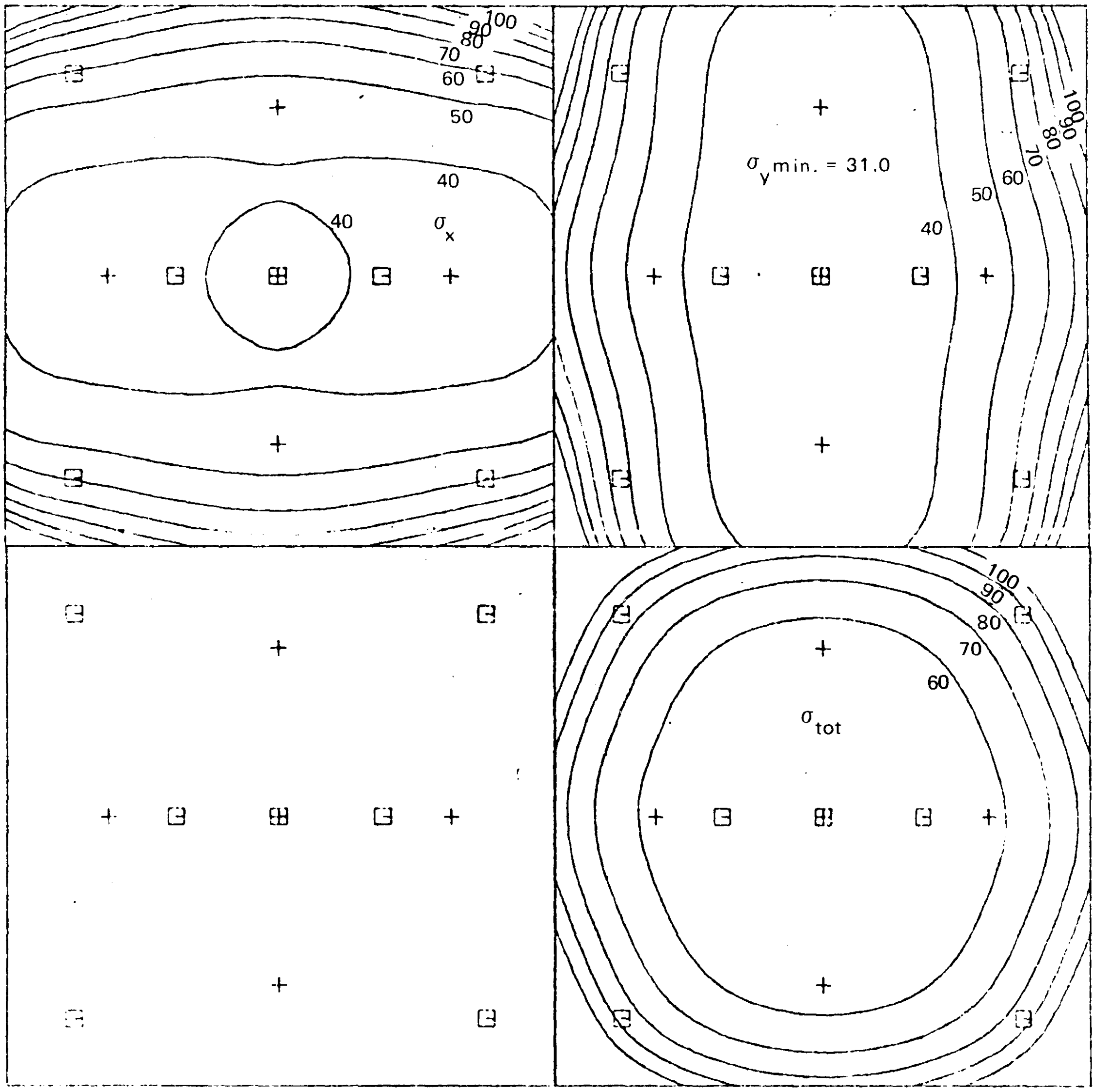


4 Layer Model
σ_v = 5%
σ_t = .005

σ_{tot}min. = 72.7

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 19

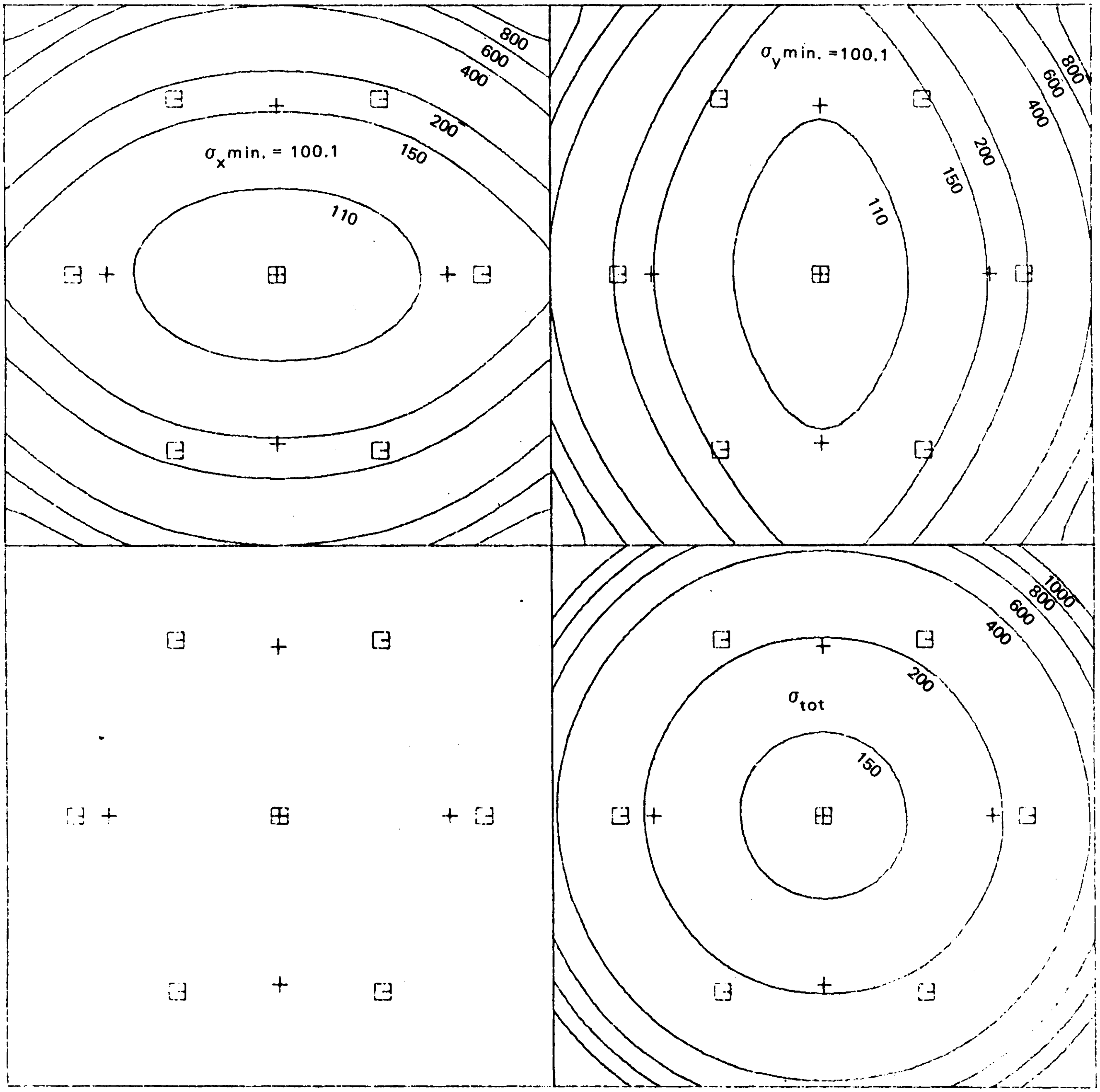


$v = 5000 + 18z$
$\sigma_v = 5\%$
$\sigma_t = .001$

σ_{tot}min. = 51.0

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 20

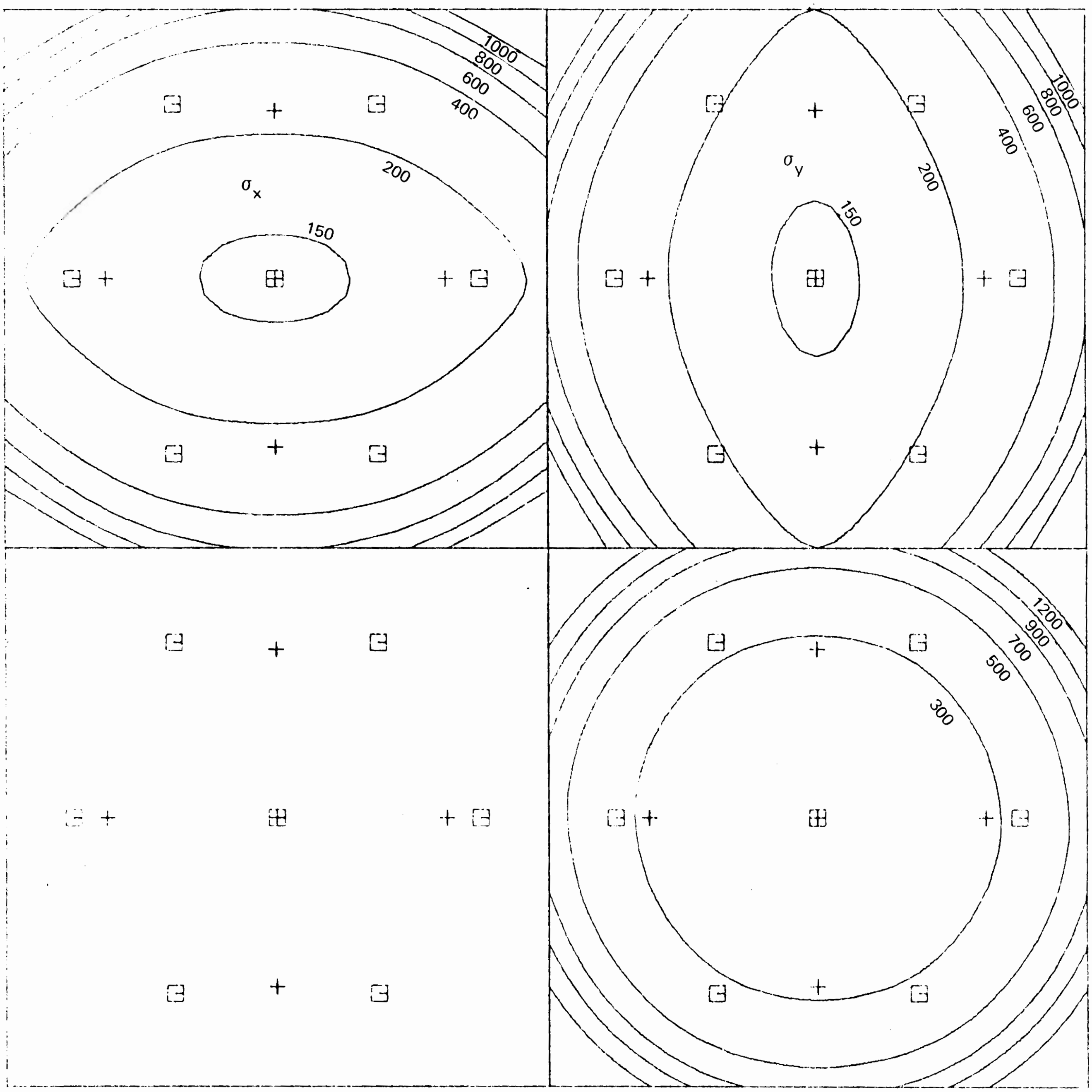


v = 5000 + 18z
σ_v = 5%
σ_t = .010

σ_{tot} min.=141.6

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 21



$v = 5000 + 18z$

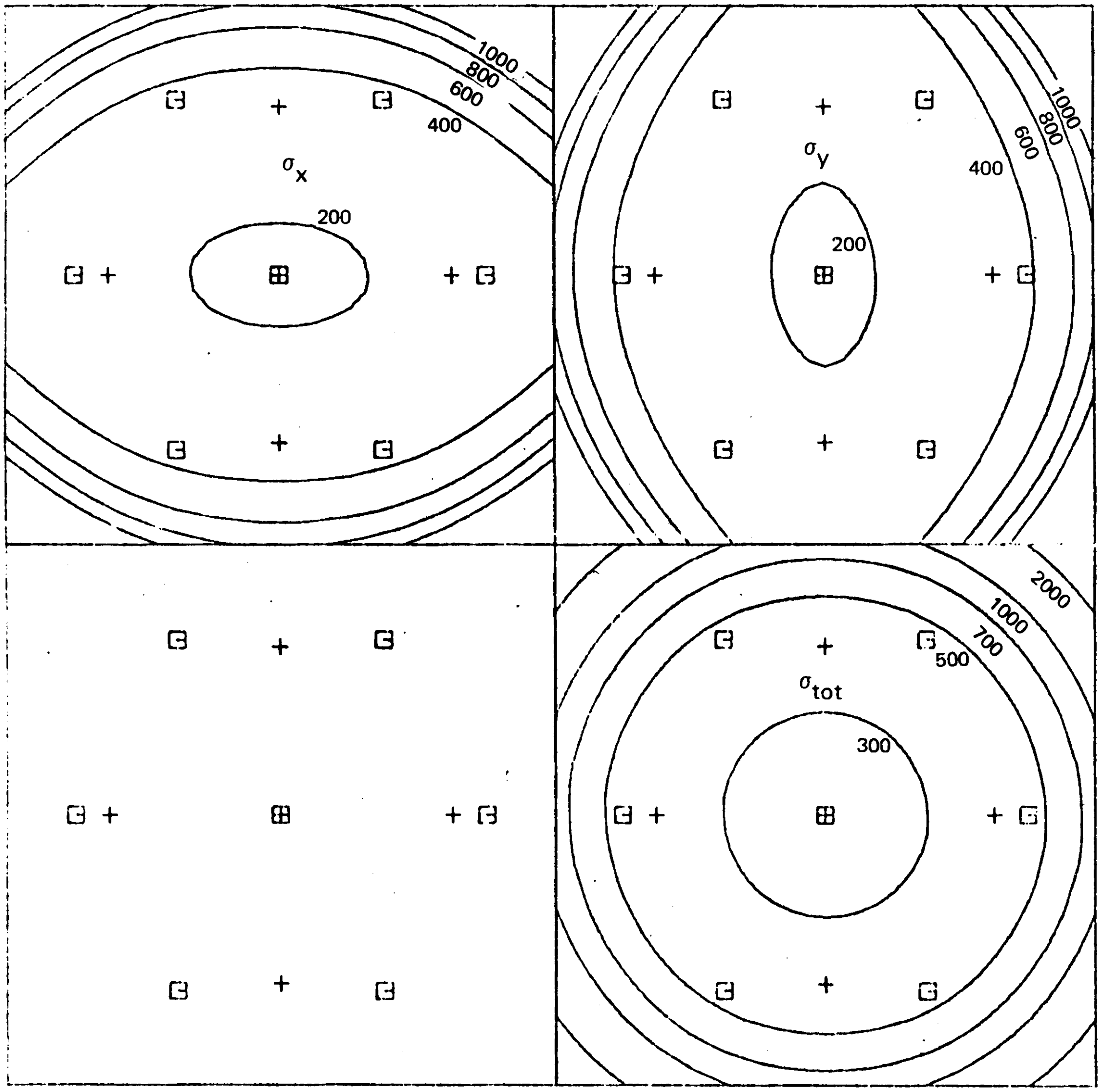
$\sigma_v = 5\%$

$\sigma_t = .015$

$\sigma_{tot}\text{min.} = 207.2$

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 22

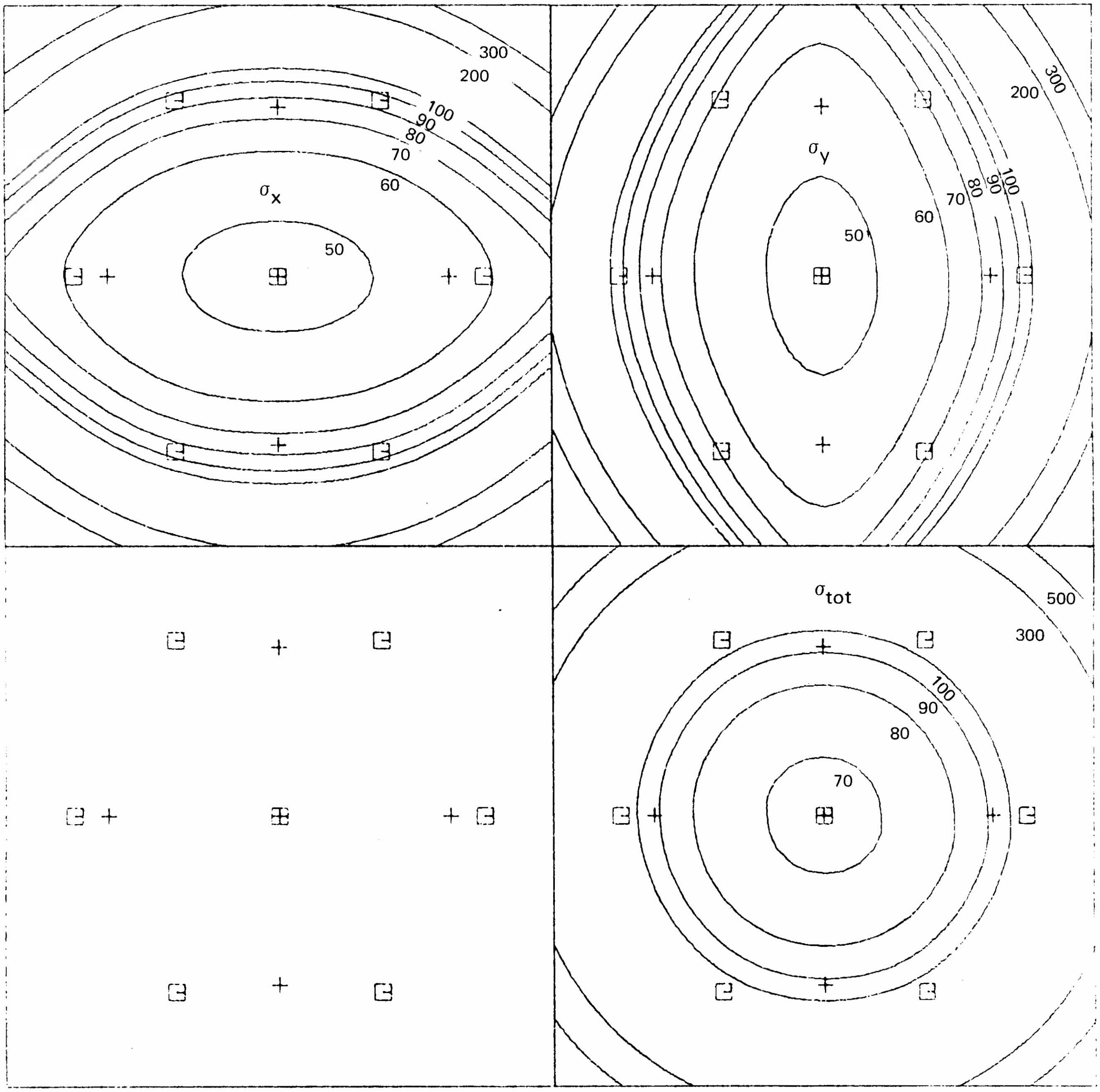


$v = 5000 + 18z$
$\sigma_v = 5\%$
$\sigma_t = .020$

σ_{tot}min. = 273.8

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET

RUN 23



v = 5000 + 18z
σ_v = 1%
σ_t = .005

σ_{tot}min. = 68.1

RUN 24

ERROR CONTOURS IN FEET
SOURCE DEPTH – 600 FEET